

# CORRECTION TO "THE STEFAN PROBLEM IN SEVERAL SPACE VARIABLES"

BY  
AVNER FRIEDMAN

The paper appeared in this Journal, **133** (1968), 51–87. We wish here to correct a few errors.

1. p. 59. The sentence beginning with "It then converges" on line 5 from the bottom is unjustified and, in any case, should be omitted. The assertion in the last sentence of this page is still true. In fact, for almost any point  $(x, t)$  of the set  $A$  where  $u=0$  we have:  $v_m(x, t) \rightarrow 0$ . This readily implies:

$$a(0-0) \leq \liminf a_m(v_m(x, t)) \leq \limsup a_m(v_m(x, t)) \leq a(0+0).$$

Since also  $a_m(v_m) \rightarrow \beta$  on  $A$ , it easily follows that  $a(0-0) \leq \beta(x, t) \leq a(0+0)$  a.e. on  $A$ .

2. p. 60. The second sentence from the top is erroneous. However, the assertion  $\delta(x, t) = c(x, u(x, t))$  a.e. on  $B$  ( $B$  the set where  $u \neq 0$ ) is still valid since  $\lim c_m(x, v_m(x, t)) = c(x, u(x, t))$  for each  $(x, t) \in B$  and since  $c_m(x, v_m) \rightarrow \delta$ .

3. p. 60. The argument leading from (2.24) to (2.25) is erroneous. Also the conclusion (2.27) is unjustified. But the assertion (2.20) of Corollary 1 is still true and can be proved as follows:

Since  $a_m(v_m(x, t)) \rightarrow a(u(x, t))$  a.e. in  $\Omega_T$ , it follows that for almost all  $\sigma \in [0, T]$   $a_m(v_m(x, \sigma)) \rightarrow a(u(x, \sigma))$  a.e. in  $G$ . The formula preceding (2.20) (on p. 60) holds also if  $\Omega_T$  is replaced by  $G \times (\sigma, T)$ , if  $\int_0^T$  is replaced by  $\int_\sigma^T$ , and if  $\int_{G(0)} a(h)$  is replaced by  $\int_{G(\sigma)} a_m(v_m(x, \sigma))$ . Taking  $m \rightarrow \infty$  we get (2.20) for all  $\sigma \in Z$ , where  $Z' \equiv [0, t] - Z$  has measure zero.

For  $s \in Z'$ , redefine  $a(u(x, s))$  as the weak limit in  $L^2(G)$  of a sequence  $\{a(u(x, \sigma_m))\}$  where  $\sigma_m \in Z$ . Call this redefined function  $\gamma(x, s)$ . Now redefine  $u(x, s)$  such that  $a(u(x, s)) = \gamma(x, s)$  if  $\gamma(x, s) < -\alpha$  or if  $\gamma(x, s) > 0$ , and  $u(x, s) = 0$  if  $-\alpha \leq \gamma(x, s) \leq 0$ . Since (2.20) holds for all  $\sigma = \sigma_m$ , it also holds for  $\sigma = s$ . This completes the proof of (2.20) (when  $u$  is redefined on the set  $G \times Z'$ ).

Since (2.26) is clearly valid, the proof of (2.21) remains unchanged.

4. p. 63. The assertion, in the second paragraph, that the sequences  $\{u_m\}$ ,  $\{c(x, u_m)\}$  and  $\{c(x, u_m)u_m\}$  are a.e. convergent is unjustified. Therefore, what is constructed is not a weak solution in the sense of the definition of p. 54, but in some other weaker sense. (The uniqueness part of Theorem 4 is valid for weak solutions in the sense of the definition of p. 54.) This result, on the existence of a weak solution, is not used anywhere later on in the paper.

NORTHWESTERN UNIVERSITY,  
EVANSTON, ILLINOIS

---

Received by the editors January 20, 1969.